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Contents

Sparse Actuator Control of Discrete-Time Linear Dynamical Systems

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ABSTRACT

This monograph presents some exciting and new results on the analysis and design of control of discrete-time linear dynamical systems using sparse actuator control. Sparsity constraints arise naturally in the inputs of several linear systems due to limited resources or the underlying physics. The monograph deals with two types of sparsity constraints: time-varying and time-invariant supported sparse control inputs. It first provides a detailed theoretical discussion on controllability under sparsity constraints, including algebraic necessary and sufficient conditions for ensuring controllability. Several related formulations, covering stabilizability, output controllability, and nonnegative controllability under sparsity constraints, are also presented. Further, for sparsely controllable systems, the monograph describes two efficient, systematic, and rigorous approaches to designing sparse control inputs: compressed sensing algorithms and spare actuator scheduling algorithms. Overall, the concepts covered in the monograph provide various sparsity models, algorithms, and analysis tools that are readily accessible to

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systems and control, signal processing, and applied mathematics readers.

1

Introduction

This section introduces the notion of sparse actuator control, convinces the reader of its importance via a few applications, and provides an overview of the content of the monograph.

1.1 What is Sparse Actuator Control?

Sparse actuator control refers to a control signal that is sparse in actuator use, i.e., we use a small subset of actuators among the available ones. In this monograph, we focus on the control of discrete-time linear dynamical systems using a few actuators.

Linear dynamical systems are well-studied and widely accepted mathematical models for describing and analyzing various control systems that evolve over time. The model serves as the core engine in diverse areas such as control systems, signal processing, communications, etc. We represent a discrete linear dynamical system using the state space model,

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k, \tag{1.1}$$

for discrete-time indices $k = 1, 2, \ldots$. Here, $\boldsymbol{x}_k$ denotes the state vector at time k. The temporal evolution of the system state is determined by the state (transition) matrix $\boldsymbol{A}$ and the input matrix $\boldsymbol{B}$. The state $\boldsymbol{x}_{k+1}$ at a given time index $k+1$ depends linearly on the previous state $\boldsymbol{x}_k$ and is influenced by the input $\boldsymbol{u}_k$ applied at time k.

Numerous practical control problems deal with the task of designing control inputs $\boldsymbol{u}_k$'s to drive the system to a desired state. Typically, the design of inputs is constrained by the energy or steady-state error requirements and the level of control stability. These problems are generally posed as (convex) optimization problems and solved using techniques like least squares. This conventional control input design utilizes all the actuators or input variables (entries of control inputs $\boldsymbol{u}_k$'s). However, several resource-aware control systems demand simpler designs where only a subset of input variables are used to control the system. Simplicity is hard to achieve and makes the design problem more challenging. Mathematically, the simplicity can be encoded using the notion of sparsity. The research area associated with this phenomenon is known as *sparse actuator control*.

A vector is said to be sparse if it contains a lot of zeros entries compared to its dimension (length). Sparse actuator control of a discrete linear dynamical system deals with control inputs having very few nonzeroes entries (or active actuators) compared to their dimension. The index set of nonzero entries of a vector is defined as its support. In this monograph, we focus on control inputs whose support set cardinality is small compared to its dimension.

The sparsity-promoting strategies considered in the literature are divided into two categories. The first strategy, called the *time-varying support case*, allows the use of different subsets of input variables at different time indices to steer the system state. In the second strategy called *time- invariant support case*, the controller identifies a subset of input variables and uses the same subset at all times to control the system state. Clearly, the second strategy is more restricted and a special case of the first strategy. Further, in both cases, it may not be able to drive the system to a desired state because of the restrictions on the control inputs. We underscore that the main difficulty here is the identification of the small subsets of inputs that can drive the system to the desired state. If the subset is known, one can ignore the columns of the input matrix corresponding to the zero entries and reduce the control design problem to the standard control design problem. The subset identification is a combinatorial problem, and the sparsity constraint is non-convex. Consequently, the analysis and design of sparse actuator control are by far not trivial. This monograph is devoted to the fundamental limits, mathematical tools, and algorithms for the sparse actuator control of linear dynamical systems.

1.2 Motivation

Sparsity constraints naturally arise in several control systems. In this section, we point to a selection of control problems that can be modeled using linear dynamical systems where the input is constrained to be sparse due to cost and energy depletion issues. The section establishes the significance of the research topic of sparse actuator control and motivates the need to study it.

1.2.1 Communication-aware Control

A networked control system refers to a large system where the controlled object(s) and the controller communicate through a communication network. For example, consider a drone controlled by a centralized ground controller. The drone sends its sensor data to the ground controller, and based on this information, the controller sends out new control commands to the drone to adjust its position, velocity, and acceleration via the network. Such communication networks are often bandwidth-limited, which motivates the use of sparse control inputs (Heemels *et al.*, 2010; Tatikonda and Mitter, 2004; Liu *et al.*, 2020; Nagahara and Quevedo, 2011). The reason for promoting sparsity is that sparse signals admit compact representations (Foucart *et al.*, 2013), leading to a lower communication burden.

1.2.2 Network Opinion Manipulation

Consider a social network where people interact with each other and influence each other's opinions on a product or idea. For example, a social network can be people living in a specific area or social media communities, where a group of people share common interests and experiences. Also, network opinion can refer to a movie rating, inclination towards a political party, or customer rating of a product. The opinion evolution over time can be represented using a linear dynamical system whose state is the network opinion (opinion of all the individuals in the network), and the state matrix models the influence of each individual's opinion on others' opinions. The network information is manipulated by external agents such as paid bloggers, social media influencers, marketing agents, etc. Their influence can be modeled as an input applied to the dynamical system that models the opinion evolution. Further, the agents are often constrained by budget (financial or physical), which can be represented using sparse inputs, where sparsity denotes the budget constraints of the agent. As an example, consider a company that sends a salesperson to market their products by offering free samples. The number of free samples is limited, and not all the samples reach the target people at the same time. The influence of such manipulators can be modeled using sparse inputs whose support denotes the individuals who receive the free sample at a given time (Joseph *et al.*, 2021).

1.2.3 Malicious Data Injection Attacks

In an electric power network, malfunctioning or compromised devices, such as power system stabilizers, generator controllers and exciters, and cyclic loads, can inject forced oscillations (0.1–15 Hz) into the network. The sources triggering these oscillations are fewer compared to the potential sources (Anguluri *et al.*, 2023; Anguluri *et al.*, 2022; Siami *et al.*, 2020). Hence, the effect of anomalous sources can be modeled as sparse control inputs to the system. Further, sparse inputs can represent data injection attacks that target a limited number of sensors in the smart grid (Cárdenas *et al.*, 2008; Hao *et al.*, 2015; Sun and Li, 2022; Chen *et al.*, 2019). Similarly, sparse inputs can represent malicious attacks on cyber-physical systems (Ma and Shi, 2022; Tsang *et al.*, 2020).

1.2.4 Efficient Control in Biological Networks

Human metabolism is generally represented by directed networks. Some example biological networks are motivated by the application of control-theoretic ideas in the analysis of biological circuits (Marucci *et al.*, 2009), biochemical reaction networks (Liu *et al.*, 2013), and systems biology (Rajapakse *et al.*, 2012). Consider a directed network where the nodes model reactions and/or metabolites. The network can be externally affected by drugs that act only on a few nodes in the network. Here, sparse inputs target to reduce the adverse side effects due to drug administration and it motivates the need to control the system using sparse inputs.

Overall, the need for sparse actuator control stems from the system's cost constraints or simply from the physics of how the system operates. Therefore, using a limited number of actuators is desirable without significantly compromising the control performance, for example, in terms of the time or energy required to reach a certain desired state.

1.3 Overview of the Monograph

This monograph is organized into five sections (excluding this introductory section) that provide a detailed study of sparse actuator control. The problem formulation and associated analysis are readily accessible to signal processing, control/systems theory, and applied mathematics communities.

Section 2 formally introduces the notion of sparse actuator control with time-varying support and defines the notion of controllability under the sparsity constraint. The central questions of the section are as follows:

> *What are necessary and sufficient conditions for ensuring controllability under sparse inputs with possibly different supports? Can we devise a computationally simple test for sparse controllability? If a system is controllable using sparse inputs, how many control input vectors are needed to drive the system from a given initial state to an arbitrary final state?*

We show that, for any linear dynamical system controllable under the sparsity constraint, a sparse actuator schedule independent of the system state exists, which can drive the system to any desired state. Further, we derive simple algebraic conditions, which are both necessary and sufficient for the sparse controllability of the system. We show that the system is sparse controllable if and only if it is controllable *and* the sparsity level exceeds the nullity of the state matrix. Unlike the more traditional Kalman-type rank tests, the derived conditions can be verified in polynomial time complexity. Finally, we characterize the time-to-control or the minimum number of input vectors required to ensure sparse controllability and show that it is bounded by the state dimension. These results form a theoretical basis for designing sparse control inputs, which we discuss in the next section.

Section 3 addresses the design of sparse control with time-varying support for a given linear dynamical system. This section seeks an answer to the following question:

> *Given a controllable linear dynamical system, how do we design sparse inputs that take the system from a given initial state to a desired final state?*

We formulate the sparse control input design in two ways. In the first approach, we formulate it as a sparse recovery problem and use the compressed sensing algorithms to solve the problem. This approach does not necessarily assume that the system is controllable, but the corresponding sparse actuator schedule depends on the initial and final states. In the second approach, we assume that the system is controllable. We design a global sparse actuator schedule that applies to any pair of initial and final states and then derive the control inputs based on the designed schedule.

Section 4 extends the idea of sparse control to other related control theory notions. The focal question addressed in the section is as follows:

> *How are the stabilizability, output controllability, and nonnegative controllability of a linear dynamical system affected by the sparsity constraints on the input?*

We show three key results in this section. The first result, perhaps surprisingly, shows that sparsity constraints do not have any effect on the stabilizability. All stabilizable systems are sparse stabilizable. The second result is on the algebraic characterization of sparse output controllability. We derive bounds on minimal sparsity levels that ensure output controllability. Finally, we show that any sparse controllable and nonnegative controllable systems are sparse nonnegative controllable. We also briefly discuss three sets of design algorithms: the first estimates sparse control inputs for system stabilization, the second estimates sparse control inputs to achieve a desired output, and the third estimates nonnegative sparse control inputs to reach a desired state.

Section 5 looks at a more stringent sparsity constraint, where all the sparse inputs have nonzero entries at the same indices. So, the section deals with the question:

> *If a linear dynamical system can be controlled using only a few actuators among the available ones, how do we choose the actuators and design the corresponding control inputs?*

We prove that the problem of finding the minimum sparsity level to make the system controllable under time-invariant support is an NP hard problem. Nonetheless, there are several approximate design algorithms that can choose a small number of actuators to control the system. There are two design approaches: one is compressed sensing algorithms that are initial and final state-dependent, and the second approach is state-independent actuator scheduling-based.

Section 6 summarizes the open problems or questions on the new area of sparse actuator control, points to the weaknesses that still have to be strengthened, and offers some concluding remarks.

Each section ends with a subsection titled "Notes" where we provide supplementary facts, additional comments, and some open questions.

Notation is usually introduced when it is used for the first time. The collection of symbols used in the text can be found on page 88.

1.4 Out of the Scope Topics

This monograph is by no means exhaustive but presents some notable recent research connecting linear dynamical systems and sparse inputs. Other important problems related to sparsity and control have been studied in the literature but are outside the scope of this monograph. We briefly discuss a few of them below.

1.4.1 Sparsity in Time or Maximum Hands-Off

An important and widely studied control paradigm related to sparsity is known as maximum hands-off control (Nagahara *et al.*, 2015). This approach is characterized by applying zero control for most of the time, resulting in minimal active periods or the shortest active duration. Since actuators remain inactive for extended periods, this method significantly reduces fuel consumption, power usage, and communication burden. It is worth noting that this strategy leverages sparsity over time, while this monograph focuses on sparsity across actuator use.

The concept of time sparsity is modeled using the ℓ_0-norm, which serves as a penalty function to measure the duration of the control signal's active support. However, the ℓ_0-norm is challenging to optimize directly and is typically approximated using its convex relaxation, the ℓ_1-norm. Their equivalence holds under an assumption called normality. Most research in this area centers on continuous-time systems, though these ideas are also extended to discrete-time models (Mai and Yin, 2023).

Sparsity methods from compressed sensing and their applicability to systems and control, covering standard sparsity methods in finite-dimensional vector spaces and optimal control methods in infinite-dimensional function spaces, has been extensively covered in Nagahara (2020) and Nagahara (2023). The idea of maximum hands-off control has been extended to general linear systems (Ikeda and Kashima, 2018; Chatterjee *et al.*, 2016; Nagahara *et al.*, 2016; Ito *et al.*, 2021), and time-varying systems (Mai and Yin, 2024). Additionally, when the normality assumption does not hold, non-convex penalty functions for promoting sparsity have also been explored in the literature (Ikeda, 2024).

1.4.2 Sparse Input Estimation From Observations

The design of sparse control inputs is closely related to the problem of sparse input estimation in linear dynamical systems. In many cases, the initial state of the system, whether sparse or non-sparse, is also unknown. By leveraging the sparsity in the system, this problem can be framed as finding the sparse solution to a linear system of equations. Various compressed sensing approaches, such as basis pursuit, sparse Bayesian learning, reweighed-ℓ_1, and reweighed ℓ_2, have been used to estimate the sparse control inputs. The algorithms and guarantees for sparse input estimation from observations, as well as other variants of the problem, are discussed in Sefati *et al.* (2015), Kafashan *et al.* (2016), Fosson *et al.* (2019), and Chakraborty *et al.* (2024) and their references.

Despite similarities and a shared compressed sensing-inspired approach to the solution, sparse control design and sparse input estimation differ significantly. In the sparse input estimation problem, the output trajectory of the system is already known, and the goal is to estimate the sparsest inputs that can drive the system along a given trajectory. In contrast, the sparse control problem does not have a predefined trajectory. Instead, the objective is to find sparse inputs while also considering system expenditures such as the energy budget, making the problems distinct.

1.4.3 Sparsity in Feedback

Another area of interest is enforcing sparsity in the controller's feedback gain matrix. Some variants of this approach focus on maximizing the number of nonzero rows in the feedback gain matrix to make the state feedback vector sparse (Polyak *et al.*, 2014; Arastoo *et al.*, 2016). This approach helps reduce the bandwidth requirement when feedback is communicated to the plant via a wireless link. Another variant aims to minimize the number of nonzero entries in the feedback matrix (Lin *et al.*, 2013; Fardad and Jovanović, 2014; Babazadeh and Nobakhti, 2016). This strategy reduces the number of communication links between the many components of large-scale and networked control systems.

1.4.4 Sparse Sensing and Sparse Observability

Since sensing problems are dual counterparts of control problems, sparse sensing or sensor scheduling represents a related area of study. In such contexts, the literature often distinguishes between two approaches: myopic strategies, which prioritize the immediate effects of selected actuators, and non-myopic strategies, which take a forward-looking perspective alongside immediate impacts (Hashemi *et al.*, 2020; Ballotta *et al.*, 2020; Vafaee and Siami, 2024).

Sensor scheduling differs significantly, as its performance metrics typically include estimation error, computational constraints, and transmission delay (especially in networked systems). In contrast, control design primarily focuses on minimizing energy usage and adhering to time-to-control constraints. Some studies have also explored joint actuator-sensor selection (Ye *et al.*, 2022), which is also not the focus here.

Another related topic includes the observability of linear systems when the initial state is sparse (Dai and Yüksel, 2013; Sanandaji *et al.*, 2014; Joseph and Murthy, 2018; Joseph and Murthy, 2019). However, sparse actuator controllability assumes a general initial state and sparse control inputs, demanding distinct analyses.

1.4.5 Minimal Input Set for Structured Systems

Certain studies have explored minimal input selection for structured systems. The primary objective is to identify a minimal cardinality set of inputs that ensures a structurally controllable network (Chapman and Mesbahi, 2013; Trefois and Delvenne, 2015). Additionally, other research has investigated methods to adjust the system configuration in order to reduce the size of the minimal input set (Abbas *et al.*, 2023; Joseph *et al.*, 2023).

While these studies address sparsity, they require graph theory-based analysis and the concept of zero-forcing sets specific to structural systems, which are not directly related to the topic of this monograph.

Interested readers are referred to the above works and references therein for extensive treatment of the topics.

1.5 Notes

Linear dynamical systems are extensively applied across numerous fields, including control systems (Zhou *et al.*, 1996), signal processing (Anderson and Moore, 2005), communications (Prasad *et al.*, 2014), economics (Brockwell *et al.*, 2002), mechanical and civil engineering (Pope III *et al.*, 2002; Shao *et al.*, 2006), and healthcare (Neumann *et al.*, 2009; Hvistendahl *et al.*, 2013). The study of linear dynamical systems with sparsity constraints dates back to 1972 (Athans, 1972). Recent research has focused on the problem of identifying sequences of sparse control inputs, for both fixed and time-varying sets of control nodes, as well as other related challenges, all of which are explored in detail in this monograph.

Several sparse control design algorithms covered in this monograph are inspired by the field of compressed sensing, particularly in Sections 3 and 5. Compressed sensing, also known as compressive sensing, compressive sampling, sparse sampling, or sparse signal recovery, is a signal processing technique that allows for efficient acquisition and reconstruction of signals by solving an underdetermined linear system. This method makes use of the principle of sparsity, enabling the recovery of signals from significantly fewer samples than traditionally required by the Nyquist-Shannon sampling theorem through optimization techniques.

Since the advent of compressed sensing theory in the early 2000s, the field has introduced a plethora of computationally efficient algorithms and sophisticated analytical tools to handle sparsity. The initial steps in this field were marked by seminal papers that combined ℓ_1-norm minimization with randomness in the measurement matrices (Candès *et al.*, 2006; Donoho, 2006). These foundational works paved the way for a robust framework that has significantly influenced various applications in signal processing and beyond. For thorough overviews of compressed sensing, refer to the works of Baraniuk (2007), Wakin *et al.* (2008), Kutyniok (2013), Fornasier and Rauhut (2015), and Foucart *et al.* (2013).

Moreover, as discussed in Section 1.2.1, sparse control offers compact representations. The advances in compressed sensing have made it possible to represent sparse vectors with fewer samples compared to non-sparse vectors, thereby reducing the communication burden. This advantage is particularly significant in systems where communication efficiency and bandwidth are critical. By minimizing the amount of data required for accurate signal representation and transmission, compressed sensing facilitates more efficient and effective control strategies in various cyber-physical applications.

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